

# EXTENDING THE USE OF THE CONFORMAL MAPPING TECHNIQUE FOR THE CALCULATION OF THE QUASI-TEM PARAMETERS OF SEVERAL CYLINDRICAL AND WRAPPED TRANSMISSION LINES

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## ABSTRACT

An approach will be outlined in order to extend the use of the conformal mapping for the calculation of the quasi-TEM parameters of several cylindrical transmission lines. The approach leads, in most cases, to analytic closed form expressions. Some of the structures which will be analyzed, have been already analyzed by using the conformal mapping, some have also been analyzed but not with the use of the conformal mapping technique, others will be analyzed here for the first time. In all cases numerical results are presented for the aid of comparisons for the previously analyzed lines or to investigate some of the properties of the new ones.

## 1. INTRODUCTION.

Analyzing various transmission lines on cylindrical dielectric materials is of great interest for the design of feed networks required for exciting microstrip patch antenna arrays on cylindrical surfaces as well as for considering the effect of wrapping in planar lines due to any severe environmental changes. Considerable work ([1] to [4] in addition to their references) has been made to calculate the design parameters of several cylindrical transmission line structures by using several methods. One of these is based on the use of the conformal mapping technique [1] so that the original cylindrical structure is transformed to a planar one whose analysis is believed to be easier. However, this approach has only been used for the analysis of single cylindrical transmission lines and it is believed that the conformal mapping technique is not suitable for use in the analysis of multiple-coupled cylindrical lines. In this contribution, an approach has been outlined in order to extend the use of the conformal mapping to the calculation of quasi-TEM parameters of several cylindrical lines. The approach leads, in most cases, to analytic closed form expressions. It should be pointed out that all the following discussions will be directed towards the circular cylindrical transmission lines, since it is possible to transfer elliptic surfaces into circular ones.

This fact is demonstrated in Figs. 1a and 1b, respectively. For example, cross-sectional ellipses in the Z-Plane (with special dimensions relations) can be transformed into cross-sectional circles in the W-Plane by using the transformation function  $W = (Z - \sqrt{Z^2 - c^2})/c$ , such that  $c = \sqrt{a_1^2 - b_1^2} = \sqrt{a_2^2 - b_2^2}$ .

## 2. OUTLINE OF THE APPROACH AND APPLICATIONS.

The approach will always start by searching for a real radial conducting, electric, or magnetic wall around which the logarithmic transformation  $T \approx \ln(W)$  is used in order to transform the interior between the two circles in the W-Plane (Fig.1b) into the interior of a rectangle in the T-Plane (Fig. 1c). It is clear that the resulting rectangle can be considered as a cross-section of a planar structure. The analysis of the resulting planar structures may then be carried out by using any of the approved methods in separate or even combined. The approach has been used to calculate the quasi-TEM parameters of several transmission lines. These will be demonstrated in the poster presentation and will include, the single thin and thick strip and microstrip lines, the two coupled thin and thick strip and microstrip lines, the single coplanar waveguide with top cover and ground plane, the single supported and inverted microstriplines, the two asymmetrical broadside-coupled microstriplines, as well as as a system of 5-striplines under an excitation for the calculation of the capacitance coefficient  $C_{33}$ . Due to the limiting space available, the use of the outlined approach will be demonstrated here for three cylindrical lines only. In both cases numerical results will be presented for the aid of comparison for the previously analyzed ones, or to demonstrate some of the properties of the new ones.

### A. Cylindrical Thin Microstrip Line.

The cross-section of the cylindrical thin microstrip line is shown in Fig. 2a. Due to the symmetry of the

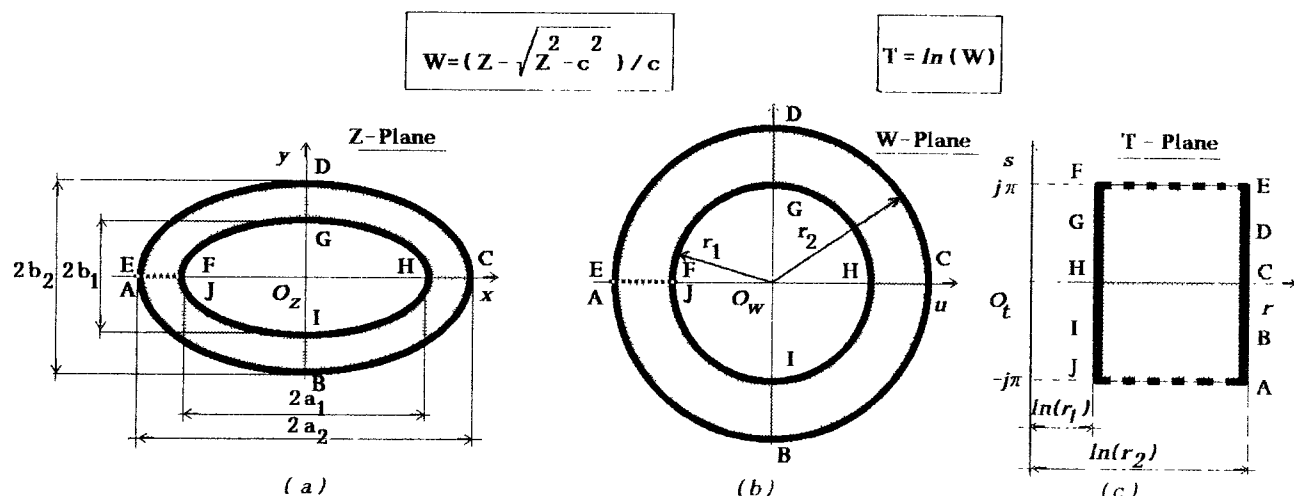


Figure 1: Transformations of an area bounded by (a) two ellipses in the Z-plane to (b) two circles in the W-plane to (c) a rectangle in the T-plane.

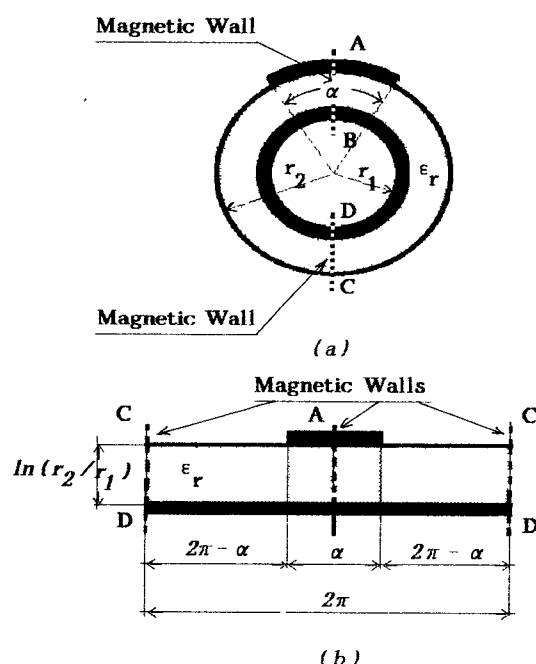


Figure 2 : Cross-sections of (a) a single cylindrical microstripline and (b) its transformed planar one.

structure, there will be two magnetic walls placed along the radial lines A-B and C-D, around which the previously mentioned logarithmic transformation can be used. The resultant transformed bounded planar microstrip structure is also shown in cross-section in Fig. 2b. It should be pointed out that the same transformation has been used by Wang [1], however, he did not recognize the presence of the lateral magnetic walls. It is true that in most practical cylindrical lines such magnetic walls are far enough from

the strip conductors such that their effect can be ignored. In practice, it is not expected that a single line will occupy the whole circumference of the cylindrical dielectric material so that its strip edges interact in a manner similar to the even-mode excited coupled lines. Moreover, this fact could be valid for a relatively small number of coupled lines. However, for cylindrical structures with multiple coupled lines, this can not always be guaranteed. Hence, it would be better to obtain the transformed planar structure with all magnetic or electric walls regardless of their relative distance from the main strip. Maintaining or omitting these walls during the analysis of the resulting planar structure, can be used in order to aid in its simplicity. This would also be very much dependent on the used method of the analysis. The planar bounded microstrip structure of Fig. 2b may be considered as a suitable candidate for many of the available numerical methods. However, closed form expressions for its quasi-TEM parameters can be derived in terms of the closed form expressions of the quasi-TEM parameters of the conventional microstripline and the coupled microstrip lines under an even-mode excitation which are available in [5] and [6], respectively. The clue to that can be seen in Fig. 3. Calculated results have shown very good agreement with those of Zeng and Wang [1]. Both results are displayed in Table 1.

## B. Cylindrical Asymmetrical Broadside-Coupled Microstriplines.

The cross-section of the cylindrical asymmetrical broadside-coupled microstriplines is shown in Fig. 4a.

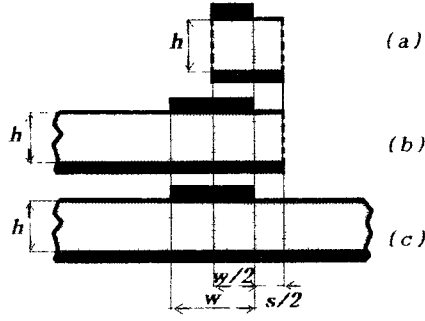


Figure 3: The clue for analyzing the transformed planar microstripline bounded by two magnetic walls ( Fig. 2b ) in terms of a single unbounded microstripline and even-mode excited coupled microstriplines.

TABLE 1: Calculated normalized  $Z_0$  obtained by the out-lined approach for the cylindrical microstrip-line of Fig. 2, with  $r_2/r_1=2$  and several values for  $\epsilon_r$  and  $\alpha$  as compared with Zeng & Wang results [1].

$\epsilon_r$	$\alpha^0/2$	$\sqrt{\epsilon} Z_0(\text{ohms})$	
		This method	Zeng & Wang
2.0	10.195	184.41	184.07
	29.883	110.81	111.71
	50.273	83.95	83.95
	100.195	52.18	51.18
6.0	10.195	201.63	202.41
	29.883	122.24	119.82
	50.273	89.17	88.23
	100.195	54.57	54.19

TABLE 2: values of  $[C_{ij}]$  of the asymmetric broad-side-coupled cylindrical microstriplines shown in Fig. 10, with  $\epsilon_{r1}=\epsilon_{r2}=\epsilon_{r3}$ ,  $r_2/r_1=r_4/r_3=1.492$  and  $r_3/r_2=1.221$ .

$\alpha_1^0$	$\alpha_2^0$	$C_{11}/\epsilon$	$C_{12}/\epsilon$	$C_{22}/\epsilon$
57.296	57.296	9.136	-5.355	9.136
57.296	45.837	8.862	-4.743	7.760
57.296	22.918	8.179	-3.064	4.807
34.377	34.377	6.134	-3.354	6.134
34.377	22.918	5.859	-2.740	4.751
34.377	11.459	5.422	-1.445	2.416

While its transformation by using the same logarithmic transformation function around the two radial magnetic walls located along the two radial lines A-B and C-D, respectively, is shown in Fig. 4b. The transformed planar bounded asymmetrical broadside-coupled microstriplines will be analyzed by using an integral equation technique. In this case, four capacitance coefficients need to be calculated, these are mainly  $C_{11}$ ,  $C_{12}$  and  $C_{22}$ . Values for these capacitance coefficients are displayed in Table 2 for typical values of the physical dimensions of the structure and  $\epsilon_{r1}=\epsilon_{r2}=\epsilon_{r3}=\epsilon$ .

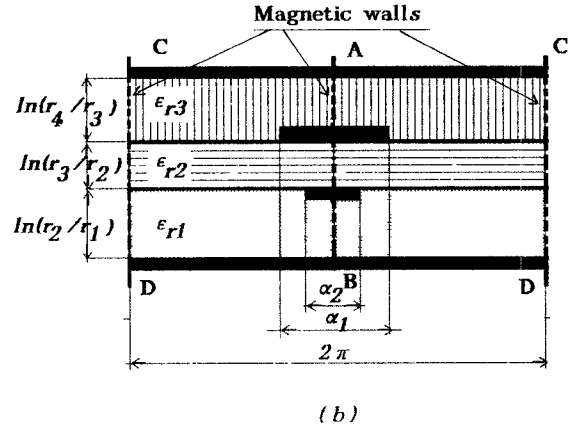
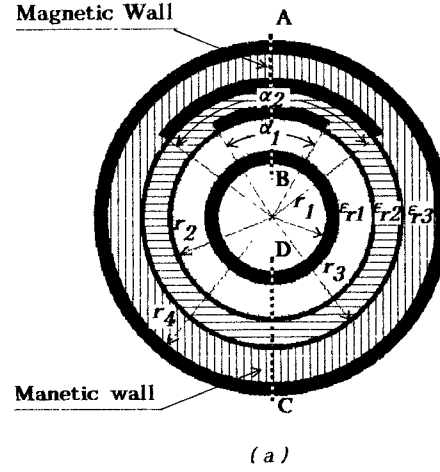


Figure 4: Cross-sections of (a) two asymmetrical broad-side-coupled cylindrical microstriplines and (b) its transformed planar coupled structure for any general mode excitations.

### C. Cylindrical 5 - Coupled Striplines.

Fig. 5a shows a cross-section in 5-coupled cylindrical striplines. Such N-coupled structure can be analyzed by calculating the coefficients of its Maxwellian capacitance matrix. It is also a common experience that the diagonal elements of this matrix are considered equal to the capacitance of the structure whenever the corresponding strip is set to a non-zero potential, while setting all other strips to zero potential. For example in order to obtain the diagonal capacitance coefficient  $C_{33}$  of the structure (self capacitance of the third line) shown in Fig. 5a, the third strip is set to non-zero potential while setting all other four strips to zero potential. In this case, magnetic walls can be easily detected along the radial lines A-B and C-D, as shown in Fig. 5a, along which the same

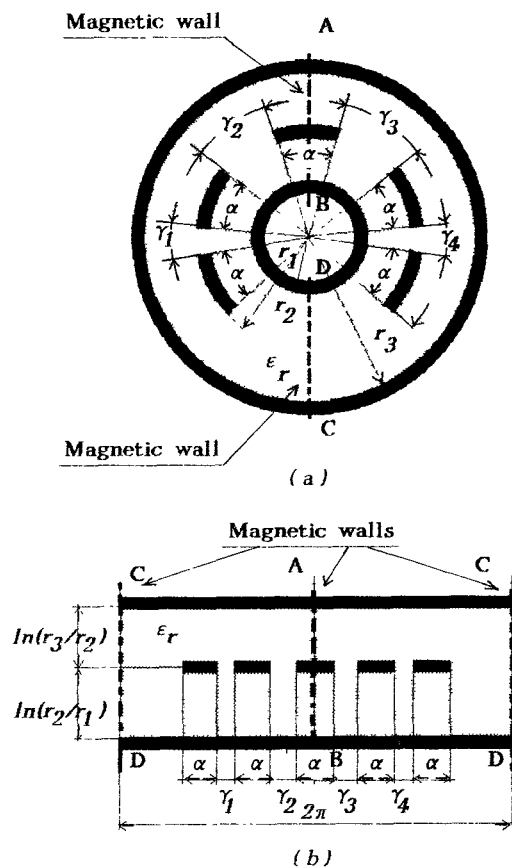


Figure 5: Cross-sections of (a) a system of symmetrical 5-coupled cylindrical striplines for the calculation of the capacitance coefficient  $C_{33}$  (self capacitance of the third conductor) and (b) its transformed planar 5-coupled striplines.

logarithmic function is used to transform the system of 5-coupled cylindrical striplines of Fig. 5a to the system of 5-coupled bounded planar striplines of Fig. 5b. As an example, for a similar system of 5-coupled cylindrical striplines with  $r_2/r_1 = r_3/r_2 = 1.649$ ,  $\alpha = 11.489^\circ$ , and  $\gamma_1 = \gamma_2 = \gamma_3 = \gamma_4 = 5.221^\circ$ , the third diagonal element of the Maxwellian capacitance matrix is calculated by using an integral equation technique and is found to be  $C_{33}/\epsilon = 3.2961$ .

### 3. CONCLUSION.

Extending the use of the conformal mapping technique for the calculation of the quasi-TEM parameters of three cylindrical transmission line structures has been demonstrated here. Due to the limited space available here, analysis of several other cylindrical transmission line structures by using the outlined approach has been

left to be demonstrated in the poster presentation. In all cases numerical results are presented for the aid of comparison for the previously analyzed structures and to investigate some of the properties of the new ones. It should be pointed out here that the application of the outlined approach will result in a planar structure which is always bounded by lateral magnetic or electric walls. In most practical cylindrical structures the effects due to the presence of such walls can be neglected. However, in some cases, presence of such walls can be used to advantage in order to simplify the analysis.

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